Lesson 9-5: Trigonometry and Area

Pulling it together

If I gave you a picture of a regular polygon, could you label the center, radius, apothem, center angle and side? Give it a go, try it with a hexagon. If you follow this link, you will find a labeled diagram to compare against.

Center angles

Of the following, are any constant (i.e. the same) for a specific type of regular polygon (for instance: all pentagons, or all octagons)?

- Center
- Radius
- Apothem
- Center angle
- Side

The only one that is constant is center angle. The center angle for an *n*-sided regular polygon is $\frac{360°}{n}$. Fill in the following table for the most common regular polygons.

Follow this link to check your work.

# sides	Polygon name	Center angle measure
3		
4		
5		
6		
8		
9		
10		
n		

Area of a regular polygon

Can you remember the formula for the area of a regular polygon? It is $A = \frac{1}{2}ap$ where a is the apothem and p is the perimeter. In chapter 7 you work extensively with this formula, but only with a few of the regular polygons. That was because you only knew how to work certain right triangles, namely 30-60-90 and 45-45-90 triangles. The center angles for triangles, squares and hexagons divide nicely into 45, 30 or 60 degree angles. This made them easy for you to work with.

Now you have some more powerful tools in your tool chest. You know how to use tangent, sine and cosine ratios. Let's put them to work for us in finding the areas of more regular polygons. I'm now going to give you a series of problems. Each will give you a different polygon with different given information. For each draw and label a diagram, determine what information you have, what you need. From that determine the trig functions that will give you the necessary information. Work through them all before you follow the given links to see the answers. Good luck!

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Given: radius

Determine the area of a pentagon with a radius of 18 *cm*. Answer.

Given: apothem

Determine the area of a hexagon with an apothem of 6 *ft*. Answer.

Given: side

Determine the area of a octagon with sides of 8 *m*. Answer.

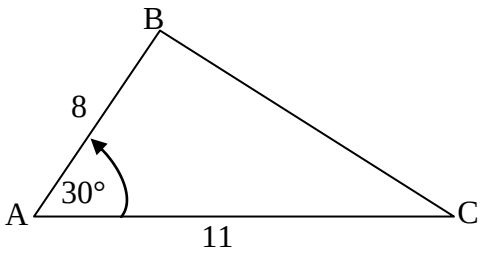
A special case – triangle

Work through the following problem. Then think about it and try to come up with a general answer/formula. The neat thing about this formula is that it will work with any triangle.

Given the following triangle with the indicated side and angle information, determine its area.

Follow this link for some hints.

Follow this link for the answer.



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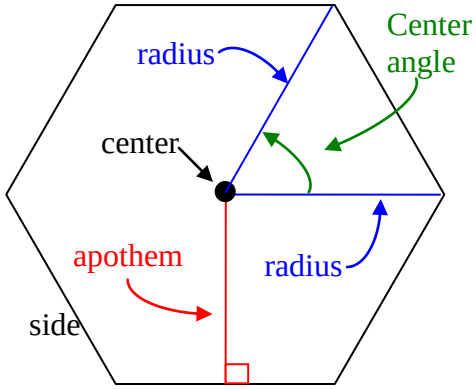
Labeled hexagon

Here is the labeled diagram of a hexagon. Check if you labeled the parts correctly!



Center angles table

Here are the center angles for the common regular polygons:

# sides	Polygon name	Center angle measure
3	Triangle	120°
4	Square	90°
5	Pentagon	72°
6	Hexagon	60°
8	Octagon	45°
9	Nonagon	40°
10	Decagon	36°
n	N-gon	$\frac{360°}{n}$

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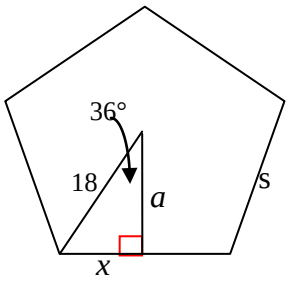
Answer: given radius

What info we know:

$n = 5$

$m\angle Center = \frac{360°}{5} = 72°$

$r = 18$ (hypotenuse)



What we need:

Apothem (adjacent side) → cosine

Perimeter…side (2 · opposite side) → sine

Answer:

$\cos 36 = \frac{a}{18};\ a = 18\cos 36 = 14.5623$

$\sin 36 = \frac{x}{18};\ x = 18\sin 36 = 10.58013;\ s = 2x = 21.16027;\ p = 5s = 105.80134$

$A = \frac{1}{2}ap = \frac{1}{2}(14.5623)(105.80134) = 770.355\,cm^2 \approx 770.4cm^2$

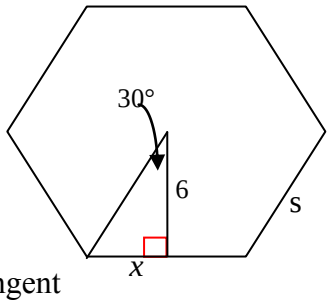
Answer: given apothem

What info we know:

$n = 6$

$m\angle Center = \frac{360°}{6} = 60°$

$a = 6$ (adjacent side)



What we need:

Perimeter…side (2 · opposite side) → tangent

Answer:

$\tan 30 = \frac{x}{6};\ x = 6\tan 30 = 3.4641;\ s = 2x = 6.9282;\ p = 6s = 41.5692$

$A = \frac{1}{2}ap = \frac{1}{2}(6)(41.5692) = 124.707\ ft^2 \approx 124.7\ ft^2$

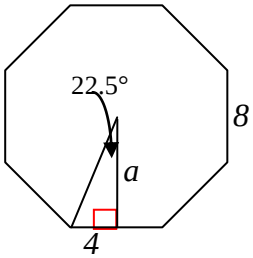
Answer: given side

What info we know:

$n = 8$

$m\angle Center = \frac{360°}{8} = 45°$

$s = 8;\ p = 8s = 64$ (opposite side – ½ s)



What we need:

Apothem (adjacent side) → tangent

Answer:

$\tan 22.5 = \frac{4}{a};\ a = \frac{4}{\tan 22.5} = 9.65685$

$A = \frac{1}{2}ap = \frac{1}{2}(9.65685)(64) = 309.019\,m^2 \approx 309.0m^2$

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Triangle area hints

- Consider $\overline{AC}$ the base.
- Construct the height of the triangle from this base.
- Can you use $\angle A$ to determine the length of the height?

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Triangle area answer

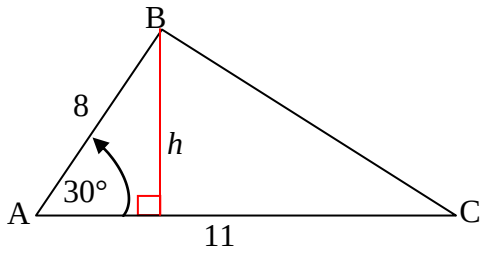
Consider $\overline{AC}$ the base and construct the height from $\overline{AC}$ to form a right triangle.

Use $\sin A$ with the hypotenuse $\overline{AB}$ and opposite side (height) to determine height.

$b = 11$

$\sin 30 = \frac{h}{8};\ h = 8\sin 30 = 4$

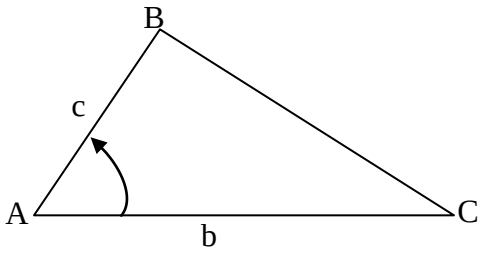
$A = \frac{1}{2}bh = \frac{1}{2}(11)(4) = 22$



General triangle area formula (Theorem 9-1)

If you have a triangle for which you know two sides and the included angle (SAS) then:

$Area = \frac{1}{2}b(c\sin A) = \frac{1}{2}bc(\sin A)$



Homework Assignment

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